REACTIVE VSWR CAUSES COMMON MODE CURRENT IN COAXIAL CABLE

A highly reactive VSWR (Voltage Standing Wave Ratio) causes common-mode current on a coaxial cable transmission line due to system imbalances arising from impedance mismatches, leading to generation of unwanted fields.

This paper offers an explanation of **Common-Mode Current** generation in a mismatched, reactively-loaded, coaxial transmission line. Definitions and an explanation is offered firstly, followed by simple mathematical proof.

1. **Impedance Mismatch and Reflections**: A highly reactive load (capacitive or inductive) reflects significant power back toward the transmitter. This reflected power creates standing waves on the coaxial cable, which produces various non-uniform impedance at various points along the cable.

2. **Field Distribution**: In a balanced and matched system, currents on the inner conductor and outer conductor of the coaxial cable are equal in magnitude but opposite in direction causing the electromagnetic fields to cancel, thus preventing the radiation of common-mode currents. However, a reactive mismatch disturbs the field symmetry causing common mode currents

3. **Imbalance in Shielding**: Due to a varying impedance caused by high **VSWR**, the current distribution on the shield of the coaxial cable (the outer conductor) becomes uneven. This can happen if the shield is not perfect, or if the grounding is not ideal. This imbalance leads to the creation of common-mode currents.

4. **Common-Mode Current Formation**: Common-mode current is the net current flowing on the outer surface of the coaxial cable shield. It is not balanced by an equal and opposite current on the inner conductor. This can happen because the voltage standing waves cause the coax shield to act as an unintentional antenna, radiating RF energy and picking up noise or signals, creating the common-mode currents.

Effects of Common-Mode Current

- **Interference and Radiation**: Common-mode currents produce unintended radiation of RF signals from the coaxial cable, causing interference with nearby electronics or other parts of the RF system.
- **Signal Degradation**: Signal quality degrades by the introduction of noise produced by unequal shield currents.

Prevention

Minimizing common-mode currents due to a high VSWR, requires a balanced, non-reactive load, good grounding, **using ferrite chokes.** Baluns (balanced to unbalanced transformers) are also commonly used in RF systems to prevent the formation of common-mode currents.

Understanding the math behind common-mode currents due to high VSWR (Voltage Standing Wave Ratio), in a coaxial cable transmission line requires dividing the problem into a few key components:

Impedance mismatch, standing waves, VSWR, and common-mode current generation. Each component is explored mathematically.

1. Impedance Mismatch and Reflection Coefficient:
The reflection coefficient (Gamma, Γ) quantifies the amount of signal reflected due to an impedance mismatch between the transmission line and the load. **A highly reactive VSWR (Voltage Standing Wave Ratio) can cause common-mode current on a coaxial cable due to an imbalance in the system arising from impedance mismatches:**

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_O} \quad [1]$$

Where:

Γ = Reflection Coefficient
Z_L = load impedance (a Complex number).
Z_0 = Coaxial cable characteristic impedance. (Usually 50Ω)

The magnitude of Γ (Gamma) tells us how much power is reflected. If Γ = 0 , no power is reflected, meaning perfect impedance matching. A non-zero Γ means standing waves will form.

2. VSWR (Voltage Standing Wave Ratio):
VSWR measures the ratio between the maximum and minimum voltages along the transmission line and is related to the reflection coefficient Γ (Gamma).

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad [2]$$

- When Γ = 0 (perfect match), VSWR = 1.
- For higher reflection coefficients, VSWR increases indicating stronger standing waves.

For a practical application assume a coaxial cable with a characteristic impedance and reactive load:

$$Z_0 = 50\,\Omega \text{ and, } Z_L = 30 + j\,40\,\Omega \quad [3]$$

In solving a reflection coefficient, first normalize the impedance:

$$Z_L = \frac{Z_L}{Z_0} = \frac{30 + j\,40}{50} = 0.6 + j\,0.8 \quad [4]$$

Now calculate Reflection Coefficient:

$$\Gamma = \frac{Z_L - 1}{Z_L + 1} = \frac{(0.6 + j\,0.8) - 1}{(0.6 + j\,0.8) + 1} = \frac{-0.4 + j\,0.8}{1.6 + j\,0.8}$$
[5]

Solving for Γ:

$$\Gamma = \frac{(-0.4 + j\,0.8)(1.6 - j\,0.8)}{(1.6 + j\,0.8)(1.6 - j\,0.8)} = -0.64 + j\,1.28 + j\,.032 - 0.64 = \frac{-1.28 + j\,1.6}{3.2} \quad [6]$$

$$\Gamma = -0.4 + j0.5 \quad [7]$$

Solving for the magnitude of $|\Gamma|$:

$$|\Gamma| = \sqrt{(-0.4)^2 + (0.5)^2} = \sqrt{(0.16+0.25)} = \sqrt{0.41} \approx 0.64 \quad [8]$$

Solving for phase shift (i.e., angle of Γ

$$\theta = \tan^{-1}\left(\frac{0.5}{-0.4}\right) = \tan^{-1}(-1.25) \approx -51.34\,° \quad [9]$$

Using formula [2]:

$$VSWR = \frac{1+0.64}{1-0.64} = \frac{1.64}{0.36} \approx 4.56 \text{ (Indicates significant load mismatch)} \quad [10]$$

3. Voltage and Current along the Transmission Line:
Standing waves are formed by the interference of incident and reflected waves.

The voltage at point z along the line is expressed *V(**x**)* as:

$$V(z) = V_o^{+}\,[e^{j\beta x} + \Gamma e^{j\beta x}] \quad [11]$$

Where:

V_o^{+} is the amplitude of the forward traveling voltage wave.

$\beta = \frac{2\pi}{\lambda}$ is the phase constant (with λ being the wavelength).

Γ is the reflection coefficient.

The magnitude of this voltage forms peaks and nulls along the transmission line, which are standing waves.

Likewise, Current at point ***x*** is written as

$$I(z) = \frac{V_o^{+¿}}{Z_0}\,[e^{j\beta x} + \Gamma e^{j\beta x}] \quad [12]$$

4. Common-Mode Current Analysis:
Calculating common-mode current induced by a reactive load mismatch is preformed by differentiating voltage change at points along the coaxial cable transmission line:

$$I_{CM} = \int_0^L \frac{dV_{(Z)}}{Z_0}\,dz \quad [13]$$

The common-mode current, I_{CM} is typically proportional to the imbalance in the system. A rough estimate for common-mode current is:

$$I_{CM} \propto I_T \cdot \Gamma \qquad [14]$$

Where:
I_T = total current on the inner conductor.
Γ = Reflection coefficient (amount of mismatch), and thus the severity of the imbalance.

If the outer conductor of the coaxial cable is not perfectly shielded or grounded, the imbalance in current between the inner and outer conductors causes part of the current to flow on the outside of the shield, leading to radiation of electromagnetic energy (common-mode radiation).

5. Radiation of Common-Mode Current:
The common-mode current flowing on the shield can radiate like an antenna, depending on the wavelength (λ) of the signal. The power radiated from this current can be approximated by:

$$P_{radiated} \approx I_{CM}^2 \cdot Z_{radiation} \qquad [15]$$

Where:
$Z_{Radiation}$ is the radiation impedance of the shield, which depends on the geometry and frequency.

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